dynamic optimisation of a three-dimensional walker

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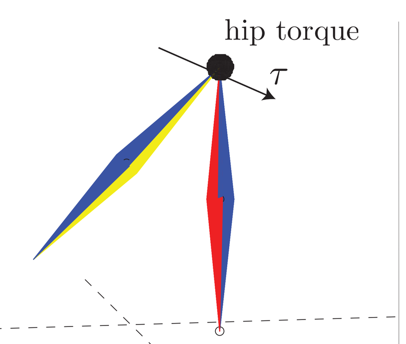
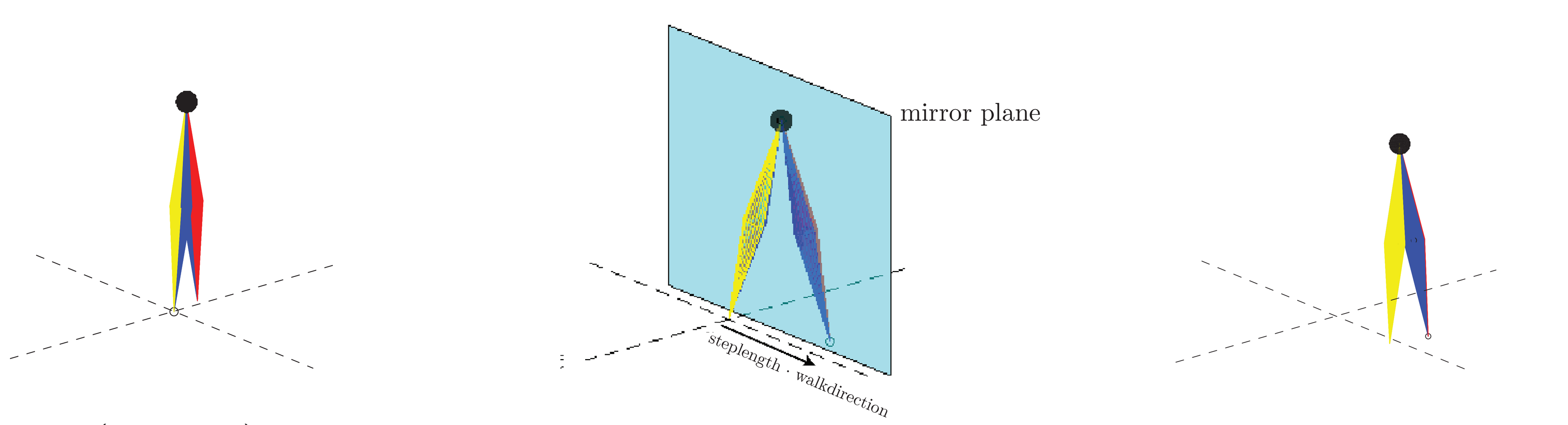
A three-dimensional compass biped is modelled as a spherical kinematic pair in which the rigid legs are combined at the hip by a spherical joint giving rise to constraints. The contact between a foot and the ground is modelled as a perfectly plastic impact, constraining the foot to stay fixed on the ground during the other leg's swing phase. This contact condition is represented by another spherical joint connection between the foot and the point of contact on the ground. During each swing phase, a variational integrator in combination with the discrete null space method for the treatment of the constraints yields the reduced forced discrete equations of motion. The contact is transferred instantaneously when the second foot hits the ground and the first one is released. Then, DMOCC (discrete mechanics and optimal control for constrained systems) is applied to determine actuating torques in the hip joint during half a gait cycle.
In the DMOCC approach, the discrete equations of motion are formulated in terms of the discrete states and controls. Together with periodic boundary conditions on the configuration and conjugate momentum, the reduced equations serve as nonlinear equality constraints for the minimisation of a specific cost functional. The algorithm yields a sequence of discrete configurations together with a sequence of actuating torques in the hip, guiding this underactuated system.

model of the walker



- rigid legs combined at hip by spherical joint

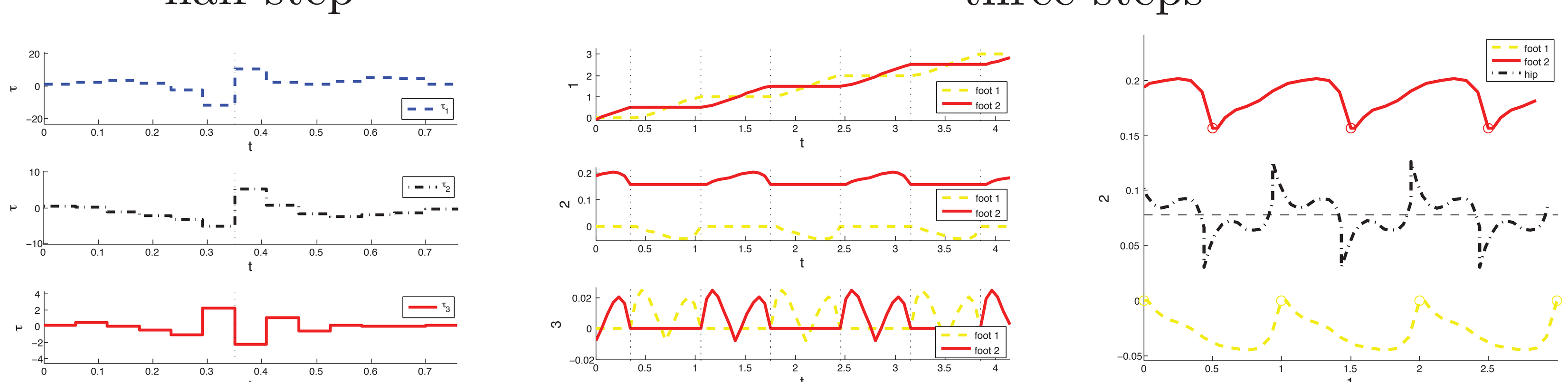
  point mass fixed at hip
  stancefoot fixed on ground by spherical joint
  actuation in hip joint by $\tau \in \mathbb{R}^3$ acts opposite on the legs
- conservation of angular momentum away from impact
- periodic boundary conditions for half step (simulated period)



$(\mathbf{q}_0, \mathbf{p}_0)$ mirror image and translation $(\mathbf{q}_N, \mathbf{p}_N)$

results

half step three steps



control torque feet feet and hip

variational discrete null space method

- $n+m$ constrained forced discrete Euler-Lagrange equations

$$D_2 L_d(\mathbf{q}_{n-1}, \mathbf{q}_n) + D_1 L_d(\mathbf{q}_n, \mathbf{q}_{n+1}) - \Delta t\, D^T \mathbf{g}(\mathbf{q}_n) \cdot \lambda_n + \mathbf{f}_{n-1}^+ + \mathbf{f}_n^- = \mathbf{0}$$
$$\mathbf{g}(\mathbf{q}_{n+1}) = \mathbf{0}$$

  in terms of n-dimensional configuration $\mathbf{q} \in \mathcal{Q}$ subject to m constraints $\mathbf{g}$ with $(n-m)$-dimensional constraint manifold $\mathcal{C} = \mathbf{g}^{-1}(\mathbf{0})$ controlled by n-dimensional forces $\mathbf{f}_{n-1}^+, \mathbf{f}_n^-$

- $n-m$ reduced forced discrete Euler-Lagrange equations

$$\mathbf{P}^T(\mathbf{q}_n) \cdot \left[D_1 L_d(\mathbf{q}_n, \mathbf{F}(\mathbf{u}_{n+1}, \mathbf{q}_n)) + D_2 L_d(\mathbf{q}_{n-1}, \mathbf{q}_n) + \mathbf{B}^T(\mathbf{q}_n) \cdot (\tau_{n-1} + \tau_n) \right] = 0$$

  $\Longleftrightarrow$ balance of momentum 'tangential to $\mathcal{C}$' $\quad \mathbf{P}^T \cdot \mathbf{p}_n^+ = \mathbf{P}^T \cdot \mathbf{p}_n^-$

  with nodal reparametrisations $\mathbf{f}_{n-1}^+ + \mathbf{f}_n^- = \mathbf{B}^T(\mathbf{q}_n) \cdot (\tau_{n-1} + \tau_n)$ and $\mathbf{q}_{n+1} = \mathbf{F}(\mathbf{u}_{n+1}, \mathbf{q}_n)$ in terms of $(n-m)$-dimensional discrete generalised incremental coordinates $\mathbf{u} \in \mathcal{U}$ and forces $\tau \in T^*\mathcal{U}$
  $\mathbf{P} : T\mathcal{U} \to T\mathcal{C}$ discrete null space matrix, $\mathbf{B}^T : T^*\mathcal{U} \to T^*\mathcal{Q}$ input transformation, $\mathbf{Q} : T^*\mathcal{Q} \to T^*\mathcal{C}$ projection

DMOCC

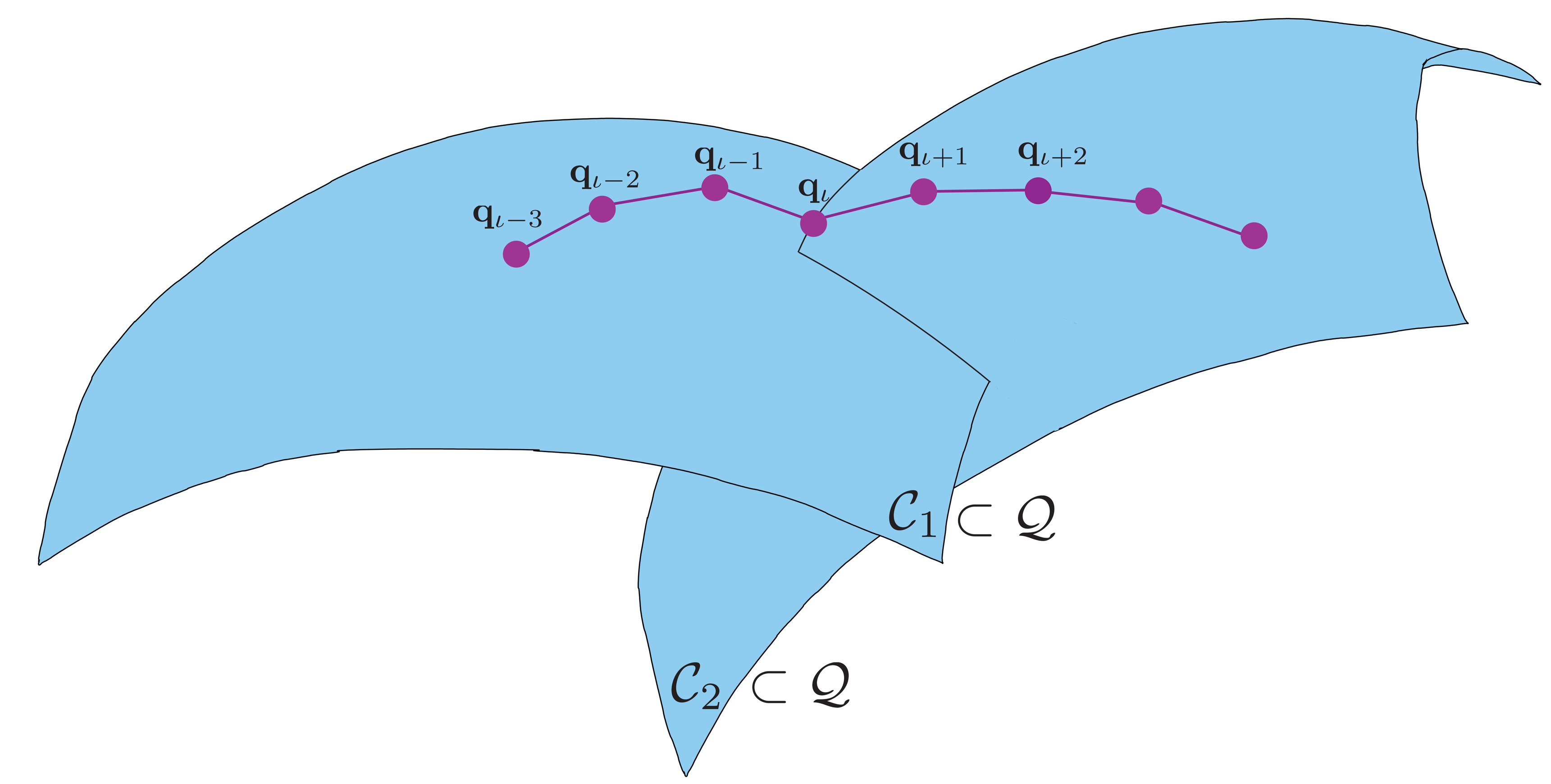
- constrained optimisation problem of minimal dimension

  minimise discrete objective function $J_d(\mathbf{u}_d, \tau_d)$ (cost of transport) wrt minimal set of unknowns $\mathbf{u}_d = \{\mathbf{u}_n\}_{n=1}^N$ and $\tau_d = \{\tau_n\}_{n=0}^{N-1}$ subject to minimal set of

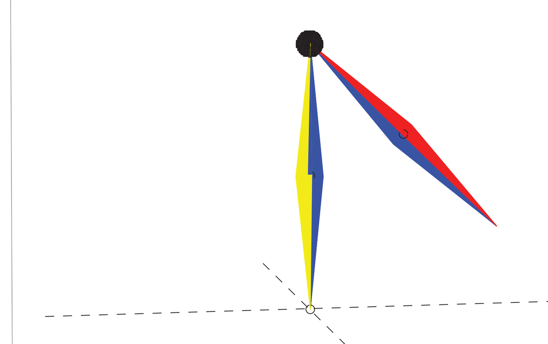
  reduced periodic boundary conditions

  reduced forced discrete Euler-Lagrange equations

transfer of contact



- $\mathcal{C}_1 = \{\mathbf{q} \in \mathcal{Q} | \text{stancefoot 1 is fixed}\}$

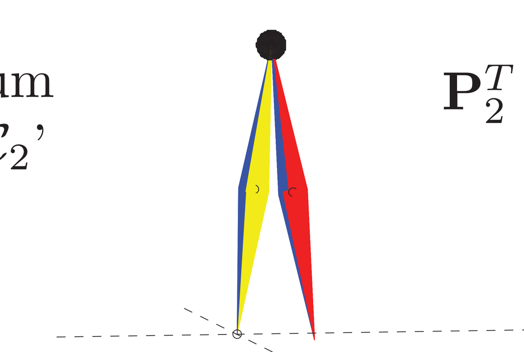


balance of momentum 'tangential to $\mathcal{C}_1$'

$$\mathbf{P}_1^T \cdot \mathbf{p}_n^+ = \mathbf{P}_1^T \cdot \mathbf{p}_n^- \qquad n = 1, \dots, \iota - 1$$

- double stance configuration $\quad \mathbf{q} \in \mathcal{C}_1 \cap \mathcal{C}_2$

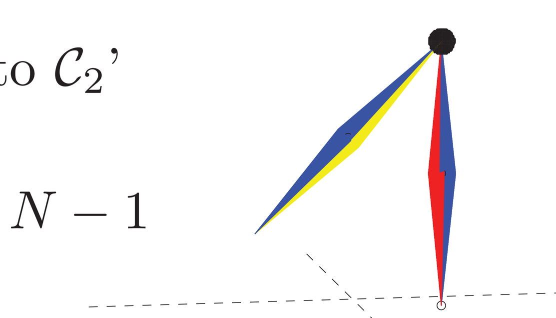
balance of momentum 'tangential to $\mathcal{C}_1 \cap \mathcal{C}_2$'



$$\mathbf{P}_2^T \cdot \mathbf{Q}_1 \cdot \mathbf{p}_\iota^+ = \mathbf{P}_2^T \cdot \mathbf{p}_\iota^-$$

- $\mathcal{C}_2 = \{\mathbf{q} \in \mathcal{Q} | \text{stancefoot 2 is fixed}\}$

balance of momentum 'tangential to $\mathcal{C}_2$'



$$\mathbf{P}_2^T \cdot \mathbf{p}_n^+ = \mathbf{P}_2^T \cdot \mathbf{p}_n^- \qquad n = \iota + 1, \dots, N - 1$$

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