optimal control of constrained multibody dynamics

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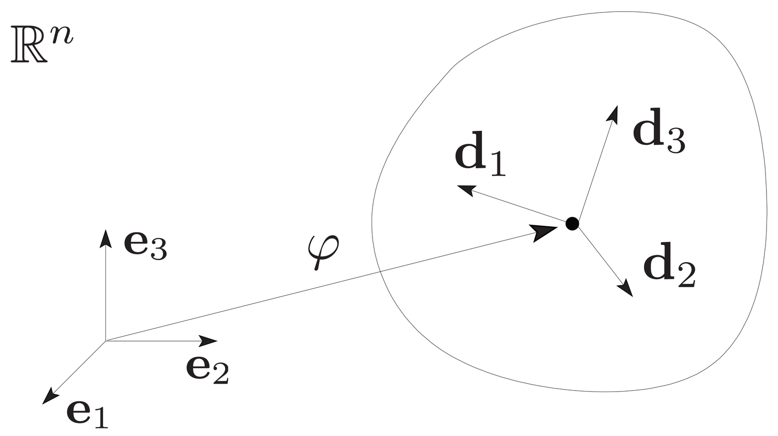
rigid body kinematics

- configuration $\mathbf{q} : [0,T] \to \mathbb{R}^n$

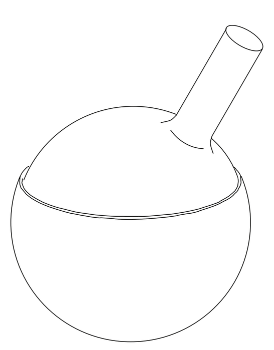
  $\mathbf{q}(t) = [\varphi, \mathbf{d}_1, \mathbf{d}_2, \mathbf{d}_3]\,(t)$



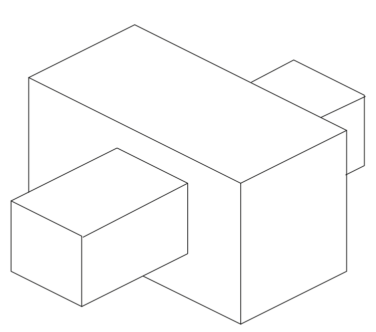
- internal constraints: orthonormal director triad

  $g_i(\mathbf{q}) = \mathbf{d}_j^T \cdot \mathbf{d}_k - \delta_{jk} = 0 \qquad j,k = 1,2,3 \qquad i = 1,\dots,m_{int}$

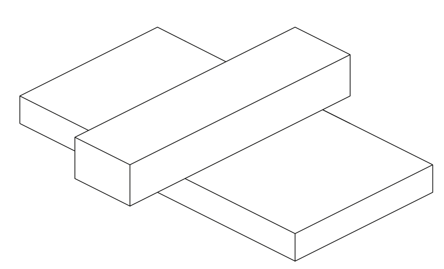
⊕ directors allow straightforward formulation of external constraints: joints in multibody systems



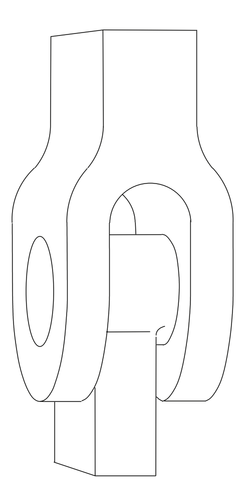
spherical $m_{ext} = 3$



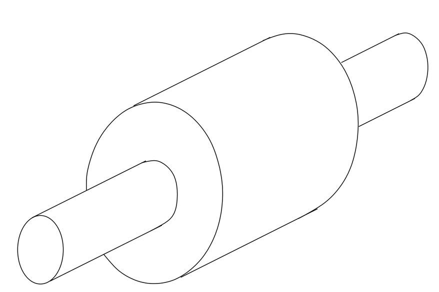
prismatic $m_{ext} = 5$



planar $m_{ext} = 3$



revolute $m_{ext} = 5$



cylindrical $m_{ext} = 4$

- holonomic constraints $\quad \mathbf{g}(\mathbf{q}) \in \mathbb{R}^m \qquad m = m_{int} + m_{ext}$

variational discrete null space method

⊕ easy formulation of discrete Lagrangian $L_d(\mathbf{q}_n, \mathbf{q}_{n+1})$ with constant mass matrix

- discrete nodal reparametrisation $\mathbf{q}_n = \mathbf{F}(\mathbf{u}_n, \mathbf{q}_{n-1})$ such that constraints are fulfilled

- discrete variational principle leads to

$$\left(\frac{\partial \mathbf{F}}{\partial \mathbf{u}_n}\right)^T \cdot \left[D_1 L_d(\mathbf{q}_n, \mathbf{q}_{n+1}) + D_2 L_d(\mathbf{q}_{n-1}, \mathbf{q}_n)\right] = \mathbf{0}$$

  to be solved for discrete generalised configuration $\mathbf{u}_{n+1}$

  ⊕ symplectic-momentum conservation
  ⊕ exact constraint fulfilment
  ⊕ minimal dimension of system: $n - m$
  ⊕ conditioning of iteration matrix is independent of Δt

unlike

- Lagrange multiplier method

  ⊖ maximal dimension of system: $n + m$
  ⊖ conditioning of iteration matrix is $\mathcal{O}(1/\Delta t^3)$

- generalised coordinates

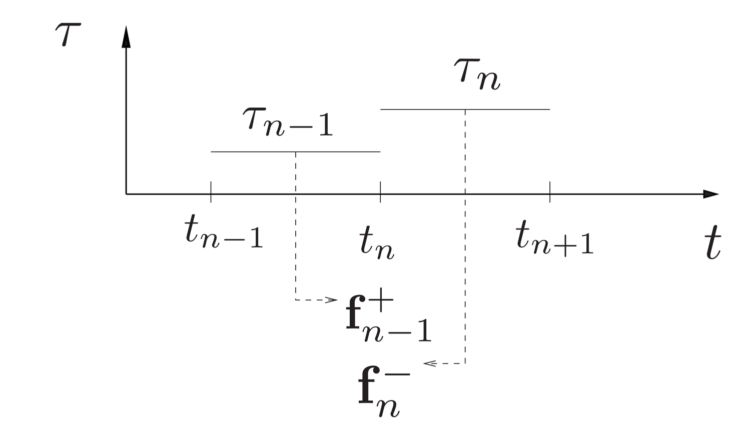
  ⊖ complicated Lagrangian, higher nonlinearity
  ⊖ difficult for multibody systems

discrete mechanics and optimal control

- discrete control forces

  $\mathbf{f}^+_{n-1}(\tau_{n-1}) \quad \mathbf{f}^-_n(\tau_n)$

  discrete generalised force τ



⊕ minimise control effort

$$J_d = \Delta t \sum_{n=0}^{N-1} ||\tau_n||^2$$

wrt minimal possible number of unknowns $\{\tau_n\}_{n=0}^{N-1}$ subject to

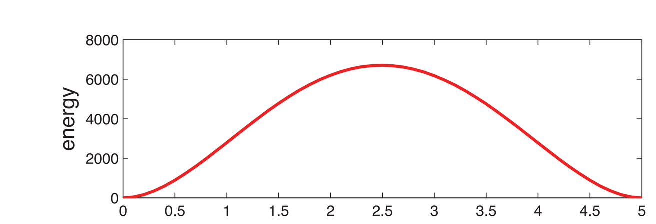
- prescribed initial and final state and

- forced time stepping equations

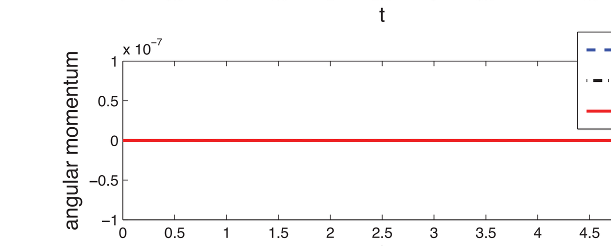
$$\left(\frac{\partial \mathbf{F}}{\partial \mathbf{u}_n}\right)^T \cdot \left[D_1 L_d(\mathbf{q}_n, \mathbf{q}_{n+1}) + D_2 L_d(\mathbf{q}_{n-1}, \mathbf{q}_n) + \mathbf{f}^+_{n-1} + \mathbf{f}^-_n\right] = \mathbf{0}$$

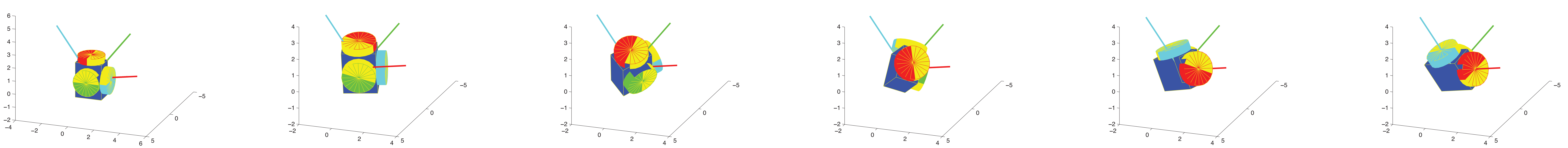
$\Longrightarrow$ optimal trajectory is consistent with dynamics
change in angular momentum exactly reflects applied torque

- manoever starts and ends at rest



- angular momentum is zero due to geometric phase





physically consistent reorientation of 'satellite' with minimal control effort

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