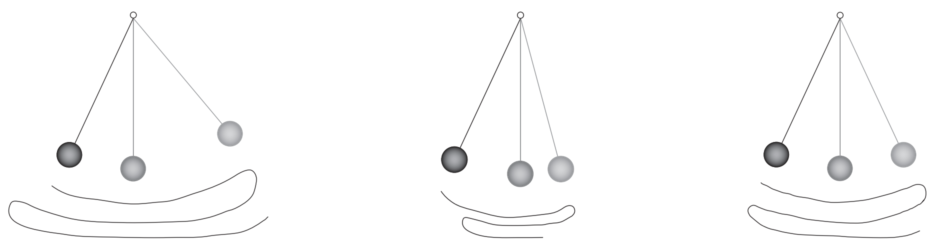
One week compact course

Discrete geometric mechanics

In this course, the Lagrangian and the Hamiltonian formulation of mechanics are presented from a geometric point of view taking into account the symplectic structure of the underlying finite dimensional state or phase manifolds, respectively. A particular focus is on Noether's theorem which relates symmetries present in the mechanical system to the conservation of momentum maps (e.g. linear and angular momentum).
Once the continuous (in time) theory has been established, a discrete formulation of Lagrangian and Hamiltonian mechanics will be presented, resulting in time-stepping schemes that inherit the geometric structure, i.e. there is a discrete symplectic form and a discrete Noether theorem. Besides the benefit of increased numerical stability, the conservation of momentum maps and symplectic structure along the discrete (approximate) trajectory enhances its veritableness since 'the unique fingerprint of the process', i.e. its 'qualitative and structural characteristics' are transferred correctly to the discrete trajectory. Different examples of such geometric integrators are discussed.



Numerical integration of the equation of motion of a pendulum. The explicit Euler scheme (left) amplifies oscillations, the implicit Euler (middle) dampens the motion, while a geometric integrator perfectly captures the periodic nature of the motion.

Addressed audience: students in semester 5 and higher.

Required previous knowledge: Calculus (Analysis) I-III.

Date and location: Monday-Friday, February 16-20, 2009, daily 10:15h-11:45h and 13:15-14:45h in Arnimallee 6 – room 031

References

J. Marsden and T. Ratiu. *Introduction to Mechanics and Symmetry. A Basic Exposition of Classical Mechanical Systems. Texts in Applied Mathematics 17*, Springer, 1994.

J. Marsden and M. West. "Discrete mechanics and variational integrators". *Acta Numerica*, pp. 357–514, 2001.

E. Hairer, G. Wanner, and C. Lubich. *Geometric Numerical Integration: Structure-Preserving Algorithms for Ordinary Differential Equations*. Springer, 2004.